$$\underbrace{a}_{x_1} + \underbrace{ar}_{x_2} + \underbrace{ar}_{x_3}{}^2 + \underbrace{ar}_{x_4}{}^3 + \cdots = \sum_{n=1}^{\infty} a r^{n-1} \qquad \frac{x_{n+1}}{x_n} = r$$

Exemplos: 1)

$2{,}317171717\ldots \in \mathbb{Q}$

$$2{,}31717\ldots = 2{,}3 + 0{,}017 + 0{,}00017 + 0{,}0000017 + \cdots$$

$$= \frac{23}{10} + \frac{17}{10^3} + \frac{17}{10^5} + \frac{17}{10^7} + \frac{17}{10^9} + \cdots$$

$$= \frac{23}{10} + \underbrace{\sum_{n=1}^{\infty} \frac{17}{10^3}\left(\frac{1}{10^2}\right)^{n-1}}_{\text{geométrica}} \quad |r| = \left|\frac{1}{10^2}\right| < 1$$

$$\frac{\frac{17}{10^9}}{\frac{17}{10^7}} = \frac{\cancel{17}}{10^9} \cdot \frac{10^7}{\cancel{17}} = \frac{1}{10^2}$$

$$\frac{\frac{17}{10^7}}{\frac{17}{10^5}} = \frac{\cancel{17}}{10^7} \cdot \frac{10^5}{\cancel{17}} = \frac{1}{10^2}$$

$$= \frac{23}{10} + \frac{17/10^3}{1 - 1/10^2}$$

$$= \frac{23}{10} + \frac{17/10^3}{\frac{10^2 - 1}{10^2}}$$

$$= \frac{23}{10} + \frac{17}{10^3} \cdot \frac{10^2}{99} = \frac{23}{10} + \frac{17}{990} = \frac{2277 + 17}{990} = \frac{2294}{990}$$

$$= \frac{1147}{495}$$

2) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \cdots$

Por fraç. parciais: $\displaystyle\frac{1}{n(n+1)} = \frac{1}{n} + \frac{-1}{n+1}$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots +$$

$$S_n = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cancel{\cdots} + \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} \xrightarrow{n\to\infty} 1 \quad \text{(série telescópica)}$$

3) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ (harmônica)

$1 > \frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \cdots$ parcelas decrescentes.

$$\underbrace{1 + \frac{1}{2}}_{} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{} + \underbrace{\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16}}_{}$$

$$> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$1 + \frac{1}{2}$

$\frac{1}{3} + \frac{1}{4} > \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$

$\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} > \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{2}$

$$\sum_{n=1}^{\infty} \frac{1}{n} > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots \quad \text{divergente.}$$

$$S_2 = 1 + \frac{1}{2}$$

$$S_{2^2} = S_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} > 1 + \frac{1}{2} + \frac{1}{2} = 1 + 2 \cdot \frac{1}{2}$$

$$S_{2^3} = S_8 > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + 3 \cdot \frac{1}{2}$$

$$S_{2^4} = S_{16} > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + 4 \cdot \frac{1}{2}$$

$$S_{2^5} = S_{32} > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + 5 \cdot \frac{1}{2}$$

$$S_{2^n} > 1 + n \cdot \frac{1}{2} \xrightarrow{n \to \infty} \infty$$

<u>Teorema</u>: Se $\sum_{n=1}^{\infty} x_n$ é convergente, então $\lim_{n\to\infty} x_n = 0$.

$$\left(\lim_{n\to\infty} \frac{1}{n} = 0, \quad \text{mas} \quad \sum_{n=1}^{\infty} \frac{1}{n} \text{ é divergente} \right)$$

<u>Teste da Divergência</u>: Se $\lim_{n\to\infty} x_n \neq 0$ ou não existir, então $\sum_{n=1}^{\infty} x_n$ é divergente.

$$\text{Dem} \Longrightarrow \text{MS}$$

$$\text{Dem} \overset{?}{\Longleftarrow} \text{MS}$$

$$\tilde{n}\,\text{Dem} \Longleftarrow \tilde{n}\,\text{MS}$$

Exemplo: $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ é divergente, pois

$$\lim_{n\to\infty} \frac{n^2}{5n^2+4} \cdot \underbrace{\frac{\frac{1}{n^2}}{\frac{1}{n^2}}}_{=1} = \lim_{n\to\infty} \frac{1}{5+\frac{4}{n^2}} = \frac{1}{5} \neq 0$$

Teorema: Se $\sum x_n$ e $\sum y_n$ convergentes, então também são convergentes:

i) $\sum (x_n + y_n) = \sum x_n + \sum y_n$;

ii) $\sum (c x_n) = c \sum x_n$.

Exemplo: $\sum_{n=1}^{\infty} \left[\frac{3}{n(n+1)} + \frac{1}{2^n} \right] = 3 \sum_{n=1}^{\infty} \frac{1}{n(n+1)} + \sum_{n=1}^{\infty} \frac{1}{2^n}$

telesc. geom.

$$= 3 \cdot 1 + \frac{\frac{1}{2}}{1-\frac{1}{2}} = 3+1 = 4 .$$

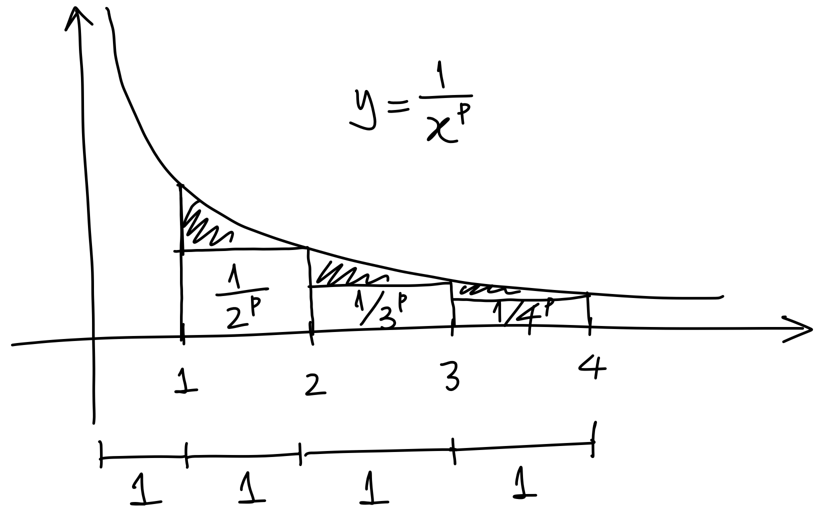
Exemplo: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (série p)

Converve se $p > 1$ e diverge se $p \leq 1$.

$p=1$: harmônica.

$p<0$: $\frac{1}{n^p} = n^{-p}$, com $-p>0 \Rightarrow \lim_{n\to\infty} \frac{1}{n^p} = \infty$

$p>0: \quad y=\dfrac{1}{x^p}$



Soma das Áreas dos retângulos $= \dfrac{1}{2^p}+\dfrac{1}{3^p}+\dfrac{1}{4^p}+\dfrac{1}{5^p}+\cdots < \displaystyle\int_1^\infty \dfrac{1}{x^p}\,dx$

$$\sum_{n=1}^{\infty}\frac{1}{n^p} = 1 + \underbrace{\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\frac{1}{5^p}+\cdots}_{\text{soma das áreas dos retângulos.}}$$ converge p/ $p>1$.

$0<p<1$

$$\frac{1}{n^p}$$